

## SPACE PHYSICS – ADVANCED OPTIONS

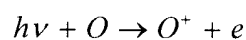
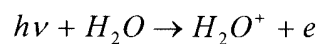
### **SOLUTIONS TO WORKSHEET ON “SOLAR WIND INTERACTION WITH PLANETARY BODIES”**

#### Question 1:

- (i) Write down the two main reactions by which cometary ions are created in the solar wind flow.
- (ii) Explain in which regions each reaction dominates.

#### Answer 1:

- (i) Cometary ions are created in the solar wind flow by photo-ionisation of cometary neutral molecules and atoms by solar EUV photons via 2 different reactions:



- (ii) The first reaction dominates in the cometary ionosphere and the second reaction dominates further away from the comet, in the solar wind flow, since in this region most of the H<sub>2</sub>O molecules have already been dissociated by solar radiation, and so a neutral coma is left of atomic O and H.

#### Question 2:

- (i) List the 4 main types of interactions that occur between the solar wind and planetary bodies.
- (ii) Describe the properties of the planetary body that cause the specific type of interaction to occur for each of the examples listed in (i).
- (iii) List some examples of solar system bodies for each of the 4 types of interactions that you listed in (i).

#### Answer 2:

- (i) There are 4 main types of interactions which occur between the solar wind and planetary bodies; these are:
  - (1) Lunar/Moon type interaction
  - (2) Earth type
  - (3) Venus type
  - (4) Comet type
- (ii) In case (1), a lunar type of interaction, this results when the planetary body does not have an intrinsic magnetic field or any atmosphere. In this case the solar wind impacts directly onto the surface of the body and no bow shock results, although a plasma wake will result downstream of the body.

For case (2) an Earth type interaction results when the planetary body has an internal magnetic field. In this instance the solar wind cannot penetrate the planetary magnetic field and is diverted around it, forming a magnetosphere.

In case (3), a Venus type of interaction occurs between the solar wind and a planetary body without an intrinsic magnetic field, but where the body has a significant atmosphere and ionosphere. The ionospheric plasma then acts as an obstacle to the solar wind flow and a bow shock results.

Case (4) results when the solar wind interacts with the nucleus of a comet, where an intrinsic magnetic field is negligible. In this instance there are two different states to

the interaction. The first state arises when the comet is inactive, that is, when the cometary nucleus is far away from the Sun and a lunar type of interaction results. When, however, the cometary nucleus is close to a Sun, a cometary coma is produced by sublimation processes and an ionosphere also forms close to the nucleus, resulting in a Venus-type interaction.

- (iii) (1) lunar type – Phobos (one of the moons of Mars), asteroids, inactive cometary nuclei  
 (2) Earth-type - Mercury, Jupiter, Saturn, Uranus, Neptune, Ganymede (one of Jupiter's moons)  
 (3) Venus type – possibly Mars and Titan (one of Saturn's moons), active cometary nuclei  
 (4) Any number of comets

Question 3:

- (i) In a planetary magnetosphere, the magnetopause separates the low- $\beta$  plasma of the magnetosphere from the high- $\beta$  plasma of the solar wind. Write down, in terms of solar wind and magnetosphere conditions, the pressure condition that holds across this magnetopause boundary.  
 (ii) Assuming we can write the field strength of the planet in terms of a dipole field, so that  $B(r) = B_{\text{dipole}} = B_{\text{int}}$ , with

$$B(r) = B_{\text{eq}} \left( \frac{R_p}{r} \right)^3$$

Where  $B_{\text{eq}}$  is the field strength at the equator on the planet's surface,  $R_p$  is the planet's radius, and  $r$  is the radial distance from the planet; derive the Chapman-Ferraro equation, taking into account the fact that the magnetopause is not just a boundary but a current layer as well and hence produces its own magnetic field. You can also assume that we can neglect the effect of the interplanetary magnetic field in the solar wind.

- (iii) Use the fact that  $M_p = B_{\text{eq}} R_p^3$ , together with values given in Table 7.1 of the Cravens book, as well as assuming the following parameters hold, to calculate the magnetopause stagnation point in the case of both the Earth and Jupiter. (Where  $M_p$  is the dipole moment of the planet,  $B_{\text{eq}}$  is the magnetic field strength at the equator at the planet's surface and  $R_p$  is the radius of the planet).  
 $u_{\text{sw}} \approx 400 \text{ km s}^{-1}$  at 1 AU and 5.2 AU  
 $n_{\text{sw}} \approx 7 \text{ cm}^{-3}$  at 1 AU and  $\approx 0.4 \text{ cm}^{-3}$  at 5.2 AU.  
 (iv) Are the values of  $R_{\text{mp}}$  obtained for the Earth and Jupiter reasonable and if not explain why this is the case.  
 (v) For Jupiter, take into account the plasma pressure effects just inside of the magnetopause assuming that the thermal pressure can be written as  $\beta p_B$ , where  $\beta \approx 5$ , and  $p_B$  is the magnetic pressure. Redo the stagnation point calculation and discuss the result in comparison to that obtained in part (iii).

Answer 3:

- (i) The magnetopause separates a low- $\beta$  plasma inside the magnetosphere with a magnetic pressure  $p_B = B^2/2\mu_0$ , where  $B$  is the magnetic field just inside the magnetopause boundary, from a high- $\beta$  plasma in the magnetosheath with a thermal pressure  $p \approx \rho_{\text{sw}} u_{\text{sw}}^2$ . The total pressure remains constant across the boundary, hence  $p = p_B$ , or

$$\rho_{\text{sw}} u_{\text{sw}}^2 = \frac{B^2}{2\mu_0} \quad (1)$$

(ii) The left hand side of equation (1) can be written as :

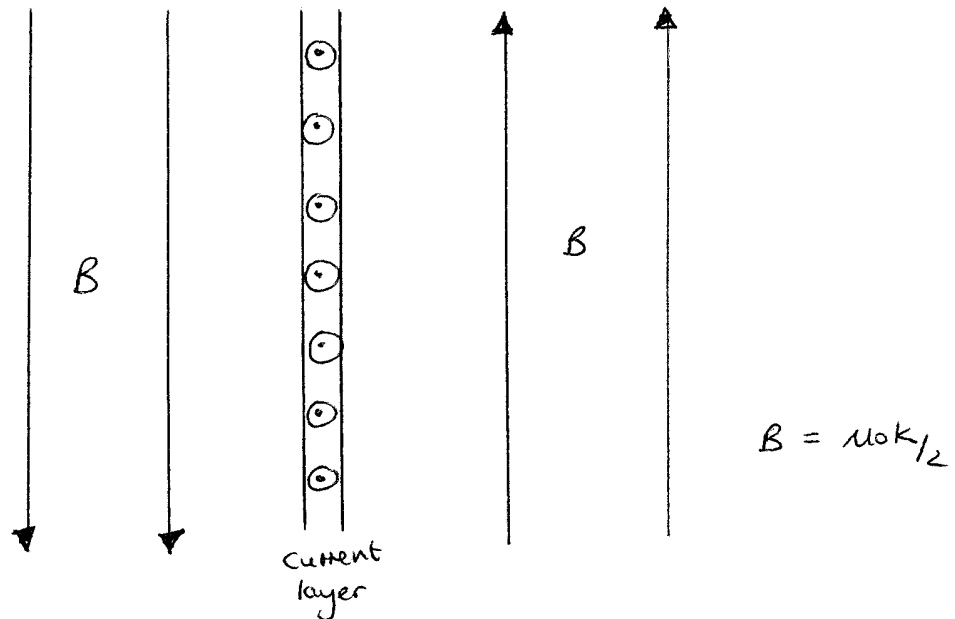
$$\rho_{sw} u_{sw}^2 = m_p n_{sw} u_{sw}^2 \quad (2)$$

Where,  $m_p$  is the proton mass,  $\rho_{sw}$  is the mass density of the solar wind and  $n_{sw}$  is the density of the solar wind.

We are told that we can assume that the planetary magnetic field is described in terms of a dipole and hence can use the fact that the magnetic field strength is given by:

$$B(r) = B_{eq} \left( \frac{R_p}{r} \right)^3 \quad (3)$$

We do however need to remember that the magnetopause boundary which separates the two regions is not just a boundary but a current layer as well, and current layers produce magnetic fields of their own and this needs to be taken account of. Assuming we have a scenario where we have an infinite current sheet carrying current per unit length  $K$  (A/m) as shown below, then Ampere's law gives that the magnetic field must be uniform on either side of the current sheet layer with opposite directions.



The field strength is given by  $B_{K(in)} = \mu_0 K / 2$  (on the right hand side of the sheet) and  $B_{K(out)} = -\mu_0 K / 2$  on the left hand side of the sheet. Hence to find the total magnetic field just inside the magnetopause boundary we need to add the effect of the magnetic field from the current layer to that of the dipole field and so

$$B_{total} = B(r) + B_{k(in)} \quad (4)$$

Now, on neglecting the IMF, the magnetic field outside of the magnetopause boundary must be zero and so  $B_{K(out)} = -B(r)$ . This then implies that  $K = 2B(r) / \mu_0$ . Hence equation (4) can be written as:

$$B_{total} = B(r) + \mu_0 K / 2 = 2B(r) \quad (5)$$

Equation (5) tells us that the magnetic field strength just inside of the magnetopause is twice the dipole value, and so it is as if the magnetic field is being compressed by the solar wind as it pushes the magnetopause towards the planet. We can now write the magnetopause force balance equation (1) as :

$$m_p n_{sw} u_{sw}^2 = \frac{B_{total}^2}{2\mu_0}$$

or

$$m_p n_{sw} u_{sw}^2 = \frac{4B_{eq}^2}{2\mu_0} \left( \frac{R_p}{R_{mp}} \right)^6$$

and hence the magnetopause distance in units of planetary radii is given by:

$$\left( \frac{R_{mp}}{R_p} \right) = \left[ \frac{2B_{eq}^2}{\mu_0 m_p n_{sw} u_{sw}^2} \right]^{1/6} \quad (6)$$

This equations shows that the magnetopause location is inversely proportional to the 1/6 power of the solar wind dynamic pressure. Larger dynamic pressure will push the magnetopause closer in towards the planet, although the (1/6) exponent implies that even large changes in solar wind dynamic pressure will only produce small changes in  $R_{mp}$ .

- (iii) It is important in this type of example to check your units, and ensure that they are consistent, since some of the parameters are given in terms of cm for example, whereas others are given in terms of km, and the final answer needs to be consistent.

Now we have that  $M_p = B_{eq} R_p^3$ , and so for the case of the Earth we have:

$M_E = 7.9 \times 10^{15} \text{ Tm}^3$ ,  $R_E = 6378 \text{ km} = 6.378 \times 10^6 \text{ m}$  and hence  $B_{eq} = 3 \times 10^{-5} \text{ T}$ , that is,

$$B_{eq} = 3 \times 10^{-5} \text{ kgs}^{-2} \text{ A}^{-1}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ Vs/Am} = 4\pi \times 10^{-7} \text{ kgms}^{-2} \text{ A}^{-2}$$

$$\rho_{sw} = m_p n_{sw} = 7 \times 1.6 \times 10^{-27} \text{ kg cm}^{-3} = 1.12 \times 10^{-20} \text{ kgm}^{-3}$$

$$u_{sw}^2 = (400 \text{ kms}^{-1})^2 = 1.6 \times 10^{11} \text{ m}^2 \text{ s}^{-2}$$

Hence

$$\left( \frac{R_{mp}}{R_p} \right)_{Earth} = \left( \frac{2 \times (3 \times 10^{-5})^2}{4\pi \times 10^{-7} \times 1.12 \times 10^{-20} \times 1.6 \times 10^{11}} \right)^{1/6}$$

$$= 9.6$$

The sub-solar magnetopause distance for the Earth with the given parameters is  $9.6R_E$ .

For the case of Jupiter we have the following:

$M_J = 1.9 \times 10^4 \times 7.9 \times 10^{15} \text{ Tm}^3 = 1.5 \times 10^{20} \text{ Tm}^3$ ,  $R_J = 71\,400 \text{ km} = 7.14 \times 10^7 \text{ m}$   
and hence  $B_{\text{eq}} = 4.1 \times 10^{-4} \text{ T}$ , that is,

$$\begin{aligned} B_{\text{eq}} &= 4.1 \times 10^{-4} \text{ kgs}^{-2} \text{A}^{-1} \\ \mu_0 &= 4\pi \times 10^{-7} \text{ Vs/Am} = 4\pi \times 10^{-7} \text{ kgms}^{-2} \text{A}^{-2} \\ \rho_{\text{sw}} &= m_p n_{\text{sw}} = 0.4 \times 1.6 \times 10^{-27} \text{ kg cm}^{-3} = 6.4 \times 10^{-22} \text{ kgm}^{-3} \\ u_{\text{sw}}^2 &= (400 \text{ kms}^{-1})^2 = 1.6 \times 10^{11} \text{ m}^2 \text{s}^{-2} \end{aligned}$$

Hence

$$\begin{aligned} \left( \frac{R_{\text{mp}}}{R_p} \right)_{\text{Jupiter}} &= \left( \frac{2 \times (4.1 \times 10^{-4})^2}{4\pi \times 10^{-7} \times 6.4 \times 10^{-22} \times 1.6 \times 10^{11}} \right)^{1/6} \\ &= 37.11 \approx 37 \end{aligned}$$

The sub-solar magnetopause distance for Jupiter with the given parameters is  $37R_J$ .

- (iv) The value of  $R_{\text{mp}}$  obtained for the Earth, that of  $9.6 R_E$  is very reasonable, since this is in fact the value which has been observed by orbiting spacecraft for average solar wind conditions. It is much more reasonable than the value which would have been obtained if we had ignored the effect of the magnetic field generated by the magnetopause current layer itself. In this instance as the notes showed a magnetopause distance of  $7.5R_E$  is derived, much less than that which is routinely observed.

However the value obtained for the magnetopause distance at Jupiter is much lower than typical distances which have been observed, with stand-off distances of between  $45 - 100 R_J$  being routinely observed at Jupiter. The reason why our calculated value of  $R_{\text{mp}}$  is so low is that the Jovian magnetosphere is in fact not entirely an Earth type. At Jupiter, although the internal magnetic field is very large, the magnetospheric plasma is also very important in the pressure balance at the magnetopause and this effect needs to be taken into account.

- (v) On taking account of the plasma pressure just inside of the magnetopause we can rewrite equation (1) as:

$$\begin{aligned} \rho_{\text{sw}} u_{\text{sw}}^2 &= [p_{\text{thermal}} + p_B]_{\text{int}} \\ &= [\beta + 1] p_{B,\text{int}} \\ &= (\beta + 1) \frac{B_{\text{int}}^2}{2\mu_0} \quad (7) \end{aligned}$$

On assuming, as previously, that the internal field is the dipole field of the planet plus the magnetic field due to the magnetopause current layer we use  $B_{\text{int}} = 2 B_{\text{dipole}}$  and using (7) we can re-derive the equation for  $R_{\text{mp}}$ , which becomes:

$$\left(\frac{R_{mp}}{R_p}\right)_{Jupiter} = \left(\frac{12 \times (4.1 \times 10^{-4})^2}{4\pi \times 10^{-7} \times 6.4 \times 10^{-22} \times 1.6 \times 10^{11}}\right)^{1/6}$$

$$= 50$$

This value of  $R_{mp}$  lies within the observed values of 45 –100  $R_J$  but can be improved yet further if we assume that currents which flow within the middle to outer Jovian magnetosphere enhance the magnetic field strength due to the planet to about  $2B_{dipole}$ . In this instance,  $B_{int} \approx 4 B_{dipole}$  (on taking magnetopause current effects into account) and the equation for  $R_{mp}$  then yields a value of  $R_{mp} \approx 63 R_J$ , which is a very reasonable value. Clearly, in the case of Jupiter, to understand the solar wind interaction with the magnetosphere it is very important to take into account the internal dynamics of the magnetosphere.

Question 4:

Using Figure 7.6 from Cravens as a basis, explain some of the major differences between the dynamics of the magnetosphere of the Earth and that of Jupiter.

Answer 4:

At Earth, the main source of energy to drive the processes in the magnetosphere arises from the solar wind, whereas Jupiter's main energy source is internal and arises from the rapid rotation of Jupiter once every 10 hours. Jupiter also has an internal plasma source, its volcanic moon Io, which generates a vast amount of plasma for the magnetosphere, and this plasma has a tendency to corotate with the planet. The rapidly rotating magnetospheric plasma is forced outwards by centrifugal forces and this causes a plasma sheet layer to form which stretches the magnetic field lines radially outwards as can be seen in the figure 7.6. This results in a much extended magnetosphere in comparison to that of the Earth.